The outermost bound orbit around a mass clump in an expanding Universe: implication on rotation curves and dark matter halo sizes

Richard Lieu [1]

[1] *Department of Physics, University of Alabama, Huntsville, AL 35899.*

ABSTRACT

Conventional treatment of cold dark matter halos employs the Navarro-Frenk-White (NFW) profile with a maximum radius set at $r = r_{200}$, where the enclosed matter has an overdensity of 200 times the critical density. The choice of $r = r_{200}$ is somewhat arbitrary. It is not the collapsed (virial) radius, but does give $r \sim 1$ Mpc for rich clusters, which is a typical X-ray size. Weak lensing measurements, however, reveal halo radii well in excess of r_{200}. Is there a surface that places an absolute limit on the extension of a halo? To answer the question, we derived analytically the solution for circular orbits around a mass concentration in an expanding flat Universe, to show that an outermost orbit exists at $v/r = H$, where v is the orbital speed and H is the Hubble constant. The solution, parametrized as r_2, is independent of model assumptions on structure formation, and *is the radius at which the furthest particle can be regarded as part of the bound system.* We present observational evidence in support of dark matter halos reaching at least as far out as $r = r_2$. An interesting consequence that emerges concerns the behavior of rotation curves. Near $r = r_2$ velocities will be biased low. As a result, the mass of many galaxy groups may have been underestimated. At $r = r_2$ there is an abrupt cutoff in the curve, irrespective of the halo profile. An important cosmological test can therefore be performed if velocity disperion data are available out to 10 Mpc radii for nearby clusters (less at higher redshifts). For Virgo it appears that there is no such cutoff.

1. Introduction

In the theory of large scale structure formation, a pillar of the standard cosmological model, one important question that has not been answered to satisfaction concerns where exactly, viz., at what threshold of overdensity, is the boundary of a mass concentration (or clump) at a given redshift. The question is pertinent to any effort in obtaining a reliable theoretical estimate of the total mass and spatial extent of the 'dark halos' in galaxies and clusters. The reason has to do with the availability, from numerical hydrodynamic

simulations (Navarro, Frenk, and White 1995, 1996, 1997; but see also the even earlier work of Dubinski & Carlberg 1991), of a 'universal' profile (known as NFW profile) for the clumps, of the form

$$\rho_m(\tilde{r}) = \frac{\delta_c \rho_c r_s^3}{\tilde{r}(\tilde{r} + r_s)^2} \tag{1}$$

for the matter density distribution, where in Eq. (1) $\tilde{r}$ denotes a *physical* radius (i.e. an invariant for bound structures), r_s is a constant scale radius, δ_c is an overdensity parameter, and ρ_c is the critical density. Since at the outer radii $\tilde{r} \gg r_s$ the density scales as $\rho_m(\tilde{r}) \sim 1/\tilde{r}^3$, *and there is no further change of functional form with increasing distance*, i.e. hierarchical structure formation codes do not seem to reveal the surface radius of a clump, the total integrated mass is divergent unless an upper radius limit, (or cutoff) R, is 'manually' assigned.

Conventionally this limit is set at $R = r_{200}$, sometimes referred also to as the virial radius, defined as the radius at which the enclosed matter density is 200 times above the critical density

$$\rho_c = \frac{3H_0^2}{8\pi G} \tag{2}$$

The choice of the virial radius R is probably just a matter of convenience, e.g. for a rich cluster like Coma one may envisage a virial mass $\sim 2 \times 10^{15}$ $M_\odot$, in which case a factor of 200 overdensity would lead, via the equation

$$M_{200} = \frac{800\pi}{3} \rho_c r_{200}^3. \tag{3}$$

to $R \approx 2.5$ Mpc, which is not far from the value believed to be 'reasonable'. (Lokas & Mamon 2003). Historically, an analytical formula for the virial radius was derived by directly appealing to the virial theorem (Lahav et al 1991). For an $\Omega_m = 0.27$ and $\Omega_\Lambda = 0.73$ cosmology (e.g. Spergel et al 2006) this radius (sometimes also called the 'collapsed' radius) is less than half the 'turnaround' radius of Peebles (1984), and corresponds to an overdensity of $\approx 90(1+z)^3$: see Fig. 1 of Eke, Cole, and Frenk (1996). As we shall see, the actual evidence points to a continuation of the NFW profile to radii well beyond this value, suggesting the already well accepted fact that particles need not be completely virialized before they become part of the clumped (or collapsed) system.

If the virial radius lacks observational significance are there other criteria available that may better connect with reality? The fundamental point here is that for a pure Newtonian (static and infinitely old) Universe there is in principle no end to the zone of gravitational influence of a clump. Clearly the same is no longer true for an expanding Universe. Can an outermost surface for a bound structure be drawn, and be subject to scrutiny? Attempts to define such a surface based upon consideration of radial motion, have been made (e.g.

Sandage 1986). The question which cannot be answered by simply calculating radial velocities, however, is whether bound orbits can still exist at these large distances. As it turns out, there *is* a radius within which circular orbits are sustainable, The derivation of this radius does not depend on any of the assumptions made about how the clump is formed.

2. Criterion for the existence of bound orbits in an expanding Universe

If in a flat and unperturbed FRW space-time with dimensionless expansion parameter $a(t)$ and $c = 1$, i.e.

$$ds^2 = dt^2 - a^2(t)(dr^2 + r^2 d\Omega^2) \tag{4}$$

we envisage particles located at constant physical distances from each other, or geometrical shapes that do not expand with the Universe, it would be more convenient to use $\tilde{r} = ar$ rather than r as radial coordinate, because our objects of interest are tied to the ar-grid and not the r-grid. Moreover, if we also perform the transformation $t \to \tilde{t}$ where $t = \tilde{t} - H\tilde{r}^2/2$, Eq. (4) will become Minkowski in form,

$$ds^2 = d\tilde{t}^2 - (1 + H^2\tilde{r}^2)(d\tilde{r}^2 - \tilde{r}^2 d\Omega^2), \tag{5}$$

apart from one correction term which is only second order in H (we assumed zero acceleration of the expansion by ignoring a qH^2 term; as will be evident from the complete treatment below, the effect of q is negligible for our purpose).

Eq. (5) is the reason why no experiments performed on earth, or within the Milky Way, can directly probe the Hubble expansion. An interesting point emerges nonetheless when one computes the radial speed of light

$$\frac{d\tilde{r}}{d\tilde{t}} = 1 - \frac{1}{2}H^2\tilde{r}^2,$$

and find that to order H^2 it decreases with increasing $\tilde{r}$, symptomatic of an effective repulsive potential that eventually prevails at large radii where expansion might overcome gravity. At this stage, however, the idea is only suggestive, because we have yet to formally include the effect of gravity.

If the large scale character of space-time is that of a flat FRW metric, but locally there is perturbation by a weak and centrally symmetric gravitational field, the line element will be modified to the form

$$ds^2 = (1 + 2\Phi)dt^2 - (1 - 2\Phi)a^2(t)(dr^2 + r^2 d\Omega^2), \tag{6}$$

where Φ is the gravitational potential (McVittie 1933, see also Futumase & Sasaki 1989). This time the transformation to coordinates $(\tilde{t}, \tilde{r})$ no longer offers a great deal of simplification. Nevertheless, McVittie (1933) showed that circular orbits maintain their constant radii

$\tilde{r} = a(t)r$, so there is still some mathematical advantage to be gained in preferring $\tilde{r}$ to r as the radial coordinate for our problem, i.e. in a $(t, \tilde{r})$ system Eq. (7) reads as

$$ds^2 = [1 + 2\Phi(\tilde{r})]dt^2 - [1 - 2\Phi(\tilde{r})][(d\tilde{r} - H\tilde{r}dt)^2 + \tilde{r}^2 d\Omega^2]. \tag{7}$$

Although in Eq. (8) one encounters a cross term of the form $H\tilde{r}d\tilde{r}dt$ which is first order in the Hubble constant, we shall soon find out when we calculate orbital behavior that, in agreement with our earlier analysis, no first order effects exist.

To compute orbits from Eq. (8), we replace each dx^μ in the generic form $ds^2 = g_{\mu\nu}dx^\mu dx^\nu$ by $dx^\mu/d\tau$ to obtain the Lagrangian for motion along the $\theta = 0$ plane as

$$2L = [1 + 2\Phi(\tilde{r})]\left(\frac{dt}{d\tau}\right)^2 - [1 - 2\Phi(\tilde{r})]\left[\left(\frac{d\tilde{r}}{d\tau} - H\tilde{r}\frac{dt}{d\tau}\right)^2 + \tilde{r}^2\left(\frac{d\theta}{d\tau}\right)^2\right], \tag{8}$$

where τ is the proper time, and a factor of two was introduced on the left side by convention (so that the canonical momenta $\partial L/\partial \dot{t} = \dot{t}$ and $\partial L/\partial \dot{r} = a^2\dot{r}$, rather than twice these quantities). The three Euler–Lagrange (or geodesic) equations corresponding to $t, \tilde{r}, \theta$ are, respectively:

$$\frac{d}{d\tau}[(1 + 2\Phi)\dot{t} + (1 - 2\Phi)H\tilde{r}(\dot{\tilde{r}} - H\tilde{r}\dot{t})] = (1 - 2\Phi)\frac{dH}{dt}\tilde{r}\dot{t}(\dot{\tilde{r}} - H\tilde{r}\dot{t}), \tag{9}$$

$$-\frac{d}{d\tau}[(1 - 2\Phi)(\dot{\tilde{r}} - H\tilde{r}\dot{t})] = \frac{d\Phi}{d\tilde{r}}[\dot{t}^2 + (\dot{\tilde{r}} - H\tilde{r}\dot{t})^2] + (1 - 2\Phi)[H\dot{t}(\dot{\tilde{r}} - H\tilde{r}\dot{t}) - \tilde{r}\dot{\theta}^2], \tag{10}$$

$$-\frac{d}{d\tau}[(1 - 2\Phi)\tilde{r}^2\dot{\theta}] = 0, \tag{11}$$

where the 'dot derivatives' are w.r.t. the proper time; e.g. $\dot{t} = dt/d\tau$. Since Eq. (9) is a homogeneous function of the time derivatives, there is an immediate first integral: L is a constant. And to normalize τ to be the proper time, we take $2L = 1$.

We are interested in the effect of expansion on circular orbits around a mass concentration, so let us look for solutions in which (at least on time scales short compared with the Hubble time) $\tilde{r}$ is a constant. Moreover, $\dot{\theta}$ and $\dot{t}$ are constants, the ratio between them gives the angular velocity:

$$\omega = \frac{d\theta}{dt} = \frac{\dot{\theta}}{\dot{t}}. \tag{12}$$

Note that on the right side of Eq. (10) the quantity dH/dt is either multiplied by $\dot{\tilde{r}}$, which we have assumed is zero, or by H. Since $dH/dt = -(1 + q)H^2$ where $q = -a\ddot{a}/\dot{a}^2$ depicts the (negative) acceleration of the expansion, the product $HdH/dt \sim H^3$ and can be ignored if we are only concerned with terms of order H^2 or lower.

Under our assumptions, the left-hand sides of Eqs. (10) to (12) all vanish, as do the right-hand sides of Eqs. (10) and (12). So we are left with one equation,

$$0 = \frac{d\Phi}{d\tilde{r}}(1 - H^2\tilde{r}^2)\dot{t}^2 - (1 - 2\Phi)(H^2\tilde{r}\dot{t}^2 + \tilde{r}\dot{\theta}^2), \tag{13}$$

or, equivalently,

$$(1 - 2\Phi)(H^2 + \omega^2)\tilde{r} = (1 - H^2\tilde{r}^2)\frac{d\Phi}{d\tilde{r}} = (1 - H^2\tilde{r}^2)\frac{GM}{\tilde{r}^2} \tag{14}$$

where in the rightmost expression M denotes the *excess* mass within radius $\tilde{r}$, after the mass contribution to that region from the mean density of matter and 'dark energy' in the Universe is subtracted. Solving for the angular velocity of the circular orbit, we obtain

$$\omega^2 = \frac{1 - H^2\tilde{r}^2}{1 + \frac{2GM}{\tilde{r}}}\frac{GM}{\tilde{r}^3} - H^2. \tag{15}$$

Provided that $GM/\tilde{r} \ll 1$ and $H^2\tilde{r}^2 \ll 1$, Eq. (16) may be recast as simply

$$\omega^2 = \frac{GM}{\tilde{r}^3} - H^2, \tag{16}$$

i.e. the net effect of the Hubble expansion is to reduce the angular velocity w.r.t. its value under the scenario of pure Newtonian gravity (or more precisely a Newtonianly perturbed Minkowski line element). A remarkable feature of Eq. (17) is that it predicts the existence of an *outermost bound orbit*, which has a radius R given by

$$\frac{GM}{R^3} = H^2, \text{ or } \frac{v}{R} = H. \tag{17}$$

Eq. (18) yielded a radius smaller than that of Eq. (4).

It is also possible to define the outermost orbit in terms of an overdensity criterion. If in Eq. (18) we write $M = 4\pi R^3\rho_{\text{clump}}/3$, where ρ_{clump} is the mean *over*density within radius R, the criterion will read

$$\rho_{\text{clump}} = \frac{3H^2}{4\pi G}. \tag{18}$$

For mass clumps at $z = 0$ we have $H = H_0$, and the rightside of Eq. (19) becomes twice the critical density, $2\rho_c$. Thus we shall henceforth refer to the boundary radius of a self-gravitating mass clump as

$$r_2 = \left(\frac{GM}{H^2}\right)^{\frac{1}{3}}, \tag{19}$$

even if the terminology is obviously loose, in the sense that for $z > 0$ clumps we have $H > H_0$ and the rightside is no longer strictly equal to $2\rho_c$.

3. Observational consequences: rotation curves and the total matter budget

Although the impression one gets from a superficial perusal of the X-ray images of clusters of galaxies is that clusters extend to radii of 1 – 2 Mpc, commensurate with the value of r_{200} as defined in section 1, there are plenty of evidence pointing to the existence of massive extended halos in clusters and galaxies, i.e. the cutoff radius is more consistent with r_2 of Eq. (20) than with r_{200}. We provide examples in each case to substantiate our claim.

3.1 Clusters

A typical number for the lower mass range of rich clusters is $M = 10^{15}$ M$_\odot$. By Eq. (20) a $z = 0$ cluster has boundary radius at $r_2 \approx 10$ Mpc, whereas the same for a $z = 0.4$ cluster is $r_2 \approx 8.5$ Mpc (we adopted $h = 0.7$ throughout this subsection). Thus, whether one 'weighs' a cluster by measuring the velocity dispersion of member galaxies, or assembling a weak lensing shear map from background sources, the easier task is to target at higher redshifts: not only is a cluster's angular size smaller because of the larger distance, but also its limiting physical radius r_2 is reduced. Weak lensing observations did reveal a very extended halo for the $z = 0.395$ cluster CL0024+1654 (Kneib et al 2003). The cluster mass as quoted in this paper is $M_{200} \approx 6 \times 10^{14}$ M$_\odot$, so that one expects $r_2 > 7.2$ Mpc (most likely much larger). Indeed, the data indicated a density profile $\rho_m(r) \sim 1/\tilde{r}^{2.4}$ continuing through the instrumental sensitivity limit at $\tilde{r} \approx 5$ Mpc without any sign of cutoff.

An important test of the standard model can be conducted if rotation (velocity dispersion) curves of clusters are measured out to $\tilde{r} = r_2$. This means, by Eq. (20), that one needs data out to 10 Mpc radii for nearby clusters, smaller for higher redshift clusters. Assuming the halo profile at the outskirts has the asymptotic NFW form, then, by Eq. (17), the observed circular velocity will scale with radius as

$$v_{\text{obs}}^2 = \frac{G(M_0 + M_c \ln \tilde{r})}{\tilde{r}} - H^2 \tilde{r}^2, \tag{20}$$

where $M_c = 4\pi r_s^3 \rho_c \delta_c$ and $M_0 = M_{200} - M_c \ln r_{200}$. Thus, while pure Newtonian gravity (the first term on the right side of the equation) predicts an essentially $v_{\text{obs}} \sim 1/\sqrt{\tilde{r}}$ decline for $r_s \ll \tilde{r} \ll r_2$, for an expanding Universe as $\tilde{r} \to r_2$ there is a sharp dive towards $v_{\text{obs}} = 0$. If the halo profile is an isothermal sphere, where $\rho(\tilde{r}) \sim 1/\tilde{r}^2$, we will have a flat rotation curve (i.e. $v_{\text{obs}} = v_0$, a constant) when $H = 0$. With expansion, however, $v_{\text{obs}}^2 = v_0^2 - H^2 \tilde{r}^2$, again an abrupt cutoff at $\tilde{r} = r_2$.

For the Virgo cluster, the data for this test are either available or (with databases like the SDSS) imminently so. The fact that there is controversy over how the Local Group relates to the cluster implies galaxies as far out as 12-13 Mpc radii are still members of Virgo. Note that at these radii $H_0 r \to 1,000$ km s^{-1}. i.e. the true velocities (due to gravity

alone) have to be well in excess of $H_0 r$ in order to maintain the rotation curve against rapid decline. Thus, either Virgo has a total mass $\gg 10^{15}$ $M_\odot$, or the predicted outermost orbit does not correspond to reality. A closer look into this problem is definitely priority task.

3.2 Galaxies

Weak lensing of background sources by foreground galaxies was investigated by Hoekstra, Yee, & Gladders (2004), and Hoekstra et al (2005), who found clear signals out to at least $\tilde{r} = r_{200}$. Based upon the lensing data alone, no definitive statements could be made about what lies beyond, apart from the fact that there was no indication whatsoever of r_{200} as representating any real cutoff radius. It is possible, nonetheless, to derive an average matter density $\bar{\rho}_\mathrm{g}$ for galaxies from the Hoekstra et al observations, and to investigate whether the inclusion of extra matter between r_{200} and r_2 would lead to a revised value for $\bar{\rho}_\mathrm{g}$ that is closer to expectation.

According to Hoekstra, Yee, & Gladders (2004), the mass within $\tilde{r} = r_{200} = 139h^{-1}$ kpc is $M_{200} = 8.4 \times 10^{11} h^{-1}$ $M_\odot$ when the data for a representative galaxy were modelled with the NFW profile. From Eq. (20), we see that even without any outlying matter, the r_2 radius for $M = M_{200}$ is $r_2 \approx 1h^{-1}$ Mpc. Since, in this paper the scale radius of the NFW profile has the fitted value of $r_s \approx 16h^{-1}$ kpc, we have $\tilde{r} \gg r_s$ for all radii $r_{200} < \tilde{r} < r_2$, so that the NFW profile reduces to $\rho_\mathrm{g}(\tilde{r}) = \delta_c \rho_c r_s^3 / \tilde{r}^3$, which can readily be integrated from $\tilde{r} = r_{200}$ to $\tilde{r} = r_2$. The outcome is, of course, the mass of the remaining galactic matter halo, viz.

$$M(r_{200} \leq \tilde{r} \leq r_2) = 4\pi r_s^3 \rho_c \delta_c \ln\left(\frac{r_2}{r_{200}}\right). \tag{21}$$

By using the best-fit δ_c parameter of $\delta_c = 2.4 \times 10^4$ as quoted in the paper, we then deduce that $M(r_{200} \leq \tilde{r} \leq r_2)$ equals ~ 85 % of $M(\tilde{r} < r_{200})$. It is clear also that in reality the halo boundary has radius $> 1h^{-1}$ Mpc because of the extra mass from the $r_2 > \tilde{r} > r_{200}$ region which we did not take into account when calculating r_2. Thus we arrive at the comparison

$$M(\tilde{r} < r_{200}) \approx M(r_{200} \leq \tilde{r} \leq r_2) \tag{22}$$

for NFW galaxy halo profiles, i.e. the inclusion of outlying matter would usually lead to a *doubling of the total mass.*

In order to derive $\bar{\rho}_\mathrm{g}$, it is necessary to combine galaxy mass and luminosity measurements, and to obtain a luminosity density for the same sample. The former was done in Hoekstra et al (2005), which still reported a representative value of M_{200} close to that quoted in the previous paragraph, and which also determined an average B band mass-to-light ratio of $M_{200}/L \approx 60\,h$ $M_\odot/L_\odot$ (see the bottom left plot of Figure 8 of Hoekstra et al 2005). The latter is to be extracted from the CNOC2 survey of Lin et al (1999), which targeted exactly

the same range of galaxies as those of Hoekstra et al (2005) and Hoekstra, Yee, & Gladders (2004). From Table 3 of Lin et al (1999), the total B band luminosity density for $0.25 < z < 0.4$ galaxies (same redshift interval as that of the Hoekstra et al 2005 lens sample) is $\rho_L^B = 1.2h \times 10^{20}$ W Hz^{-1} Mpc^{-3}. Since the B band solar luminosity is $L_\odot^B = 2.19 \times 10^{11}$ W Hz^{-1}, we may now couple this with the aforementioned mass-to-light ratio to arrive at an average mass density of

$$\bar{\rho}_g \approx 3.29 \times 10^{10} h^2 \text{ M}_\odot \text{ Mpc}^{-3}. \tag{23}$$

When comparing with the critical density of $6.11 \times 10^{11} h^2$ M$_\odot$ Mpc^{-3} at $z = 0.3$, the mean redshift of the Hoekstra et al (2005) lenses, we see that the galaxies account for 5.4 % of the matter at $z = 0.3$.

Is this percentage reasonable? From the matter budget analysis of Fukugita (2004) and Fukugita, Hogan, & Peebles (1998) emerges the picture that ~ 50 % of the matter in the near Universe is still 'missing', and may well reside in galaxies and their extended halos. If $\Omega_m \approx 0.27$, then the expectation is $\Omega_g \approx 13.5$ %, i.e. $\sim$ twice the percentage value as our 5.4 %, which of course was inferred from the galaxy mass-to-light ratio where mass refers to M_{200}. Given however, that we demonstrated a doubling of a galaxy's mass when the matter between $\tilde{r} = r_{200}$ and $\tilde{r} = r_2$ is included (i.e. Eq. (22)), the conclusion of a very extended halo component of baryons and dark matter fulfilling the anticipation of Fukugita (2004) would appear in order.

3.3 Groups

Groups of galaxies are the 'dark horse', in the sense that they pose a major systematic uncertainty to the matter budget of the near Universe, also in the direction of *raising* the fraction of matter in halos. The difference from clusters and galaxies is that, while the former harbors negligible fraction of Ω_m and the latter ~ 50 % of Ω_m if halos are included, groups can potentially be the refuge for a great deal more matter than either of them. Lieu & Mittaz (2005) analyzed the ESO survey database of 1,168 nearby groups (Ramella et al 2002) and found, at $h = 0.7$, a number density of 1.56×10^{-4} Mpc^{-3} and a mean virial mass of $\bar{M} \approx 1.15 \times 10^{14}$ M$_\odot$ per group. This leads, without inclusion of any extra mass that may be in halo extensions, to a group matter density of $\Omega_m/2$. Moreover, there is another effect. In the ESO survey, the peak velocity dispersion is $\sigma \approx 70$ km s^{-1}, with a corresponding virial mass of $M \approx 10^{12.5}$ M$_\odot$ (see Figure 1 of Lieu & Mittaz 2005). The radius at which σ applies is therefore at $\tilde{r} \approx GM/(2\sigma^2) = 2.66$ Mpc, so that $H_0\tilde{r}$ for $h = 0.7$ is not far below the circular velocity $v = \sqrt{2}\sigma$, i.e. we are again in a regime where, by Eq. (17), the observed velocity (hence inferred mass) for these groups is biased low by the Hubble effect on the orbits. Thus, however one looks at them, groups have a tendency to become 'heavier', which is why some authors (e.g. Guimaraes, Myers, & Shanks 2005) believe that the matter content of groups

is a number to be reckoned with.

In conclusion, the chief emphasis of this paper concerns the derivation of r_2, and the observational evidence for its significance, including the estimate that galaxies can account for the missing 50 % of the WMAP matter density at low redshifts if the halo mass between $r = r_{200}$ and $r = r_2$ is counted. An important test of the standard model is to search for the decline of cluster velocity dispersions near $\tilde{r} = r_2$. The model will be challenged if this decline is *not* seen out to radii $r > r_2$, as seems to be the case for Virgo.

References

Dubinski, J., & Carlberg, R.G. 1991, ApJ, 378, 496.

Eke, V.R., Cole, S., & Frenk, C. 1996, MNRAS, 282, 263.

Fukugita, M., 2004, in IAU Symp. 220, Dark matter in galaxies, ed. S.D. Ryder et al. (San Francisco: ASP), 227.

Fukugita, M., Hogan, C.J., & Peebles, P.J.E., 1998, ApJ, 503, 518.

Futumase, T. & Sasaki, M. 1989, Phys Rev D, 40, 2502.

Guimaraes, A.C.C., Myers, A.D., & Shanks, T. 2005, MNRAS, 362, 657.

Hoekstra, H., Yee, H.K.C., & Gladders, M.D., 2004, ApJ, 606, 67.

Hoekstra, H., Hsieh, B.C., Yee, H.K.C., Lin, H., & Gladders, M.D., 2005, ApJ, 635, 73.

Kneib, J. -P., Hudelot, P., Ellis, R.S., Treu, T., Smith, G.P., Marshall, P., Czoske, O., Smail, I., & Natarajan, P., 2003, ApJ, 598, 804.

Lahav, O., Lilje, P. B., Primack, J. R., & Rees, M. J., 1991, MNRAS, 251, 128.

Lieu, R., & Mittaz, J.P.D., 2005, ApJ, 628, 583.

Lin, H., Yee, H. K. C. Carlberg, R. G., Morris, S. L., Sawicki, M., Patton, D.R., Wirth, G., & Shepherd, C.W., 1999, ApJ, 518, 533.

Lokas, E.L, & Mamon, G.A. 2003, MNRAS, 343, 401.

McVittie, G.C., 1933, MNRAS, 93, 325.

Navarro, J.F., Frenk, C.S., & White, S.D.M. 1995, MNRAS, 275, 56.

Navarro, J.F., Frenk, C.S., & White, S.D.M. 1996, ApJ, 462, 563.

Navarro, J.F., Frenk, C.S., & White, S.D.M. 1997, ApJ, 490, 493.

Peebles, P.J.E., 1984, ApJ, 284, 439.

Ramella, M., Geller, M.J., Pisani, A., & da Costa, L.N., 2002, AJ, 123, 2976.

Ramella, M. et al, 1999, A & A, 342, 1.

Sandage, A. 1986, ApJ, 307, 1.

Spergel, D., et al 2006, ApJ, submitted.